

FIRST SEMESTER MSC MATHEMATICS ASSIGNMENT QUESTIONS

Differential Equations (MM213)

Assignment Questions

- 1) Find the general solution of $(2x^2+2x)y''+(1+5x)y'+y=0$ at $x=0$
- 2) Determine the nature of the point $x=\infty$ for Legendre's equation $(1-x^2)y''-2xy'+p(p+1)y=0$
- 3) Prove that $j_{1/2}(x)=\sqrt{(2/\pi x)}\cos x$
- 4) Eliminate the arbitrary function F from the equation and find the corresponding pde of $z=x+y+F(x,y)$
- 5) Find the general integral of $yzp+xzq=x+y$
- 6) Solve by Charpits method $x^2p^2+y^2q^2=4$
- 7) Find the indicial equation and its roots for the differential equation $X^2y''+Xy'+(x^2-1)y=0$
- 8) Reduce $U_{xx}-x^2U_{yy}=0$ to a canonical form.

TOPOLOGY - 1 (MM214)

1. Suppose X is a metric space, A and B are subsets of X then
 - a) Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$
 - b) Prove that $\overline{A \cap B} = \overline{A} \cap \overline{B}$
2. Verify that Z is nowhere dense in \mathbb{R}
3. Verify that Q is first category in \mathbb{R}
4. Suppose X is a metric space. Then prove that the following are equivalent
 - a) X is second countable.

b) X is separable.

c) X is Lindelof.

5. Prove that any open interval (a, b) is homeomorphic to $(0, 1)$

6. Prove that if X and Y are topological spaces and $A \subset X$ and $f : X \rightarrow Y$ is continuous then $f|_A : A \rightarrow Y$ is continuous when A has the subspace topology. Prove the converse of the result is not true.

7. If a space X is connected and locally pathwise connected, prove that X is pathwise connected.

8. Show that a space is T_2 if and only if every point of the intersection of its closed neighborhoods.

Linear Algebra (MM 211)

1) Suppose a and b real numbers, but not both 0. Find real numbers c and d such that $\frac{1}{a + bi} = c + id$

2) Prove that $-(-v) = v$ for every $v \in V$

3) For each of the following subsets of F^3 , determine whether it is a subspace of F^3

a) $\{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 0\}$

b) $\{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 4\}$

c) $\{(x_1, x_2, x_3) \in F^3 : x_1 x_2 x_3 = 0\}$

4) Suppose U and W are subspaces of R^8 such that $\dim U = 3$ and $\dim W = 5$ and $U + W = R^8$. Prove that $U \cap W = \{0\}$

5) Prove that if T is linear map from F^4 to F^2 such that

Null $T = \{(x_1, x_2, x_3, x_4) \in F^3 : x_1 = 5x_2, \text{ and } x_3 = 7x_4\}$, then T is surjective.

6) Define $T \in L(F^3)$ by $T(z_1, z_2, z_3) = (2z_1, 0, 5z_3)$. Find all eigen values and eigen vectors of T

Suppose T is a linear map on V . Let $\{u_1, \dots, u_n\}$ and $\{v_1, \dots, v_n\}$ be basis of V . Show that

$$M(T, (\{u_1, \dots, u_n\})) = A^{-1} M(T, (\{v_1, \dots, v_n\})) A$$

7) Prove that $\det(AB) = \det A \cdot \det B$

Real Analysis –I (MM 212)

- 1) Define bounded variation and total variation of a function
- 2) Show that $V(x) = V_f(a, x)$ is continuous, if f is continuous.
- 3) Define rectifiable path and prove that $\Lambda_f(a, b) = \Lambda_f(a, c) + \Lambda_f(c, b)$
- 4) Define Riemann-Stieltjes integral and state and prove integration by parts.
- 5) State and Euler's Summation formula
- 6) State and prove Riemann condition for integrability
- 7) If $f \in R(\alpha)$ then show that $f \in R(V)$
- 8) Define uniform convergence of a series and show
- 9) State and prove Cauchy condition for uniform convergence
- 10) Prove that every continuous function on a closed bounded subset D of \mathbb{R}^2 is uniformly continuous on D
- 11) Let $\mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x, y) = \begin{cases} x + y & \text{if } x \neq y \\ 1 & \text{if } x = y \end{cases}$, prove that $f(x, y)$ does not exist at $(0, 0)$
- 12) Give an example of a function which is continuous at (x_0, y_0) but not differentiable at (x_0, y_0)
- 13) Find the partial derivative of the following
 - i) $f(x, y) = x^2 + y^2$

$$\text{ii) } f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0) \end{cases}$$

14) Find the directional derivative

$$\text{i) } f(x, y) = \begin{cases} \frac{x^2y}{x^2+y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0) \end{cases}$$
