

FUNCTIONAL ANALYSIS  
course code:MM232  
ASSIGNMENT QUESTIONS (SEMESTER 3)

1. Let  $\|\cdot\|$  and  $\|\cdot\|'$  be two norms on a linear space X. Then the norm  $\|\cdot\|$  is equivalent to the norm  $\|\cdot\|'$  if and only if there exists  $\alpha, \beta > 0$  such that  $\beta\|\cdot\|' \leq \|\cdot\| \leq \alpha\|\cdot\|'$  for all  $x \in X$ .
2. Let  $l^2$  and let  $l_n$  be defined as  $l_n(k) = \begin{cases} 1 & \text{if } k = n \\ 0 & \text{otherwise} \end{cases}$   
Show that  $l_1, l_2, l_3, l_4, \dots$  is compact.
3. Show by an example that an infinite dimensional subspace of a normed space X may not be closed in X.
4. Let  $X \neq 0$  and Y be normed space. Prove that  $BL(X, Y)$  is Banach iff Y Banach.
5. Show that any two norms on a finite dimensional linear space are equivalent.
6. Show that a Banach space cannot have a countably infinite basis.
7. Let X be a normed linear space and Y a closed subspace of X with  $Y \neq X$ , if  $0 < r < 1$  prove that there exist  $X_r \in X$  such that  $\|X_r\| = 1$  and  $r < d(X_r, Y) \leq 1$
8. Prove that
  - i)  $d(ax, ay) = |a|d(x, y)$
  - ii)  $d(a+x, a+y) = d(x, y)$
 where d is a metric induced by  $\|\cdot\|$  on a normed space X
9. For  $x \in C_\infty$ , let  $f(x) = \sum_{n=1}^{\infty} x(n)$  Show that f is not continuous.
10. Find the norm of the linear functional f defined by  $f(x) = \int_{-1}^0 x(t)dt - \int_0^1 x(t)dt$  where  $x \in [-1, 1]$
11. Show that a closed subspace of a Banach Space is Banach.
12. Show that the inverse of  $F : X \rightarrow Y$  is continuous bijective linear map.

ASSIGNMENT QUESTIONS

COMPLEX ANALYSIS-1

Course Code: MM 231

- 1) Let  $D = \{z: |z| < 1\}$  and find all mobius transformation T such that  $T(D) = D$ .
- 2) If  $T(z) = (az+b)/(cz+d)$ , find necessary and sufficient conditions that  $T(\Gamma) = \Gamma$ , where  $\Gamma$  is the unit circle.
- 3) show that a mobius transformation T satisfies  $T(0) = \infty$  and  $T(\infty) = 0$  iff  $Tz = az^{-1}$

4) Let  $G$  be a region and let  $f$  and  $g$  be analytic functions on  $G$  such that  $f(z)g(z) = 0$  for all  $z$  in  $G$ . Show that either  $f \equiv 0$  or  $g \equiv 0$

5) show that if  $f: G \rightarrow \mathbb{C}$  is analytic and  $\gamma$  is a rectifiable curve in  $G$  then  $f \circ \gamma$  is also a rectifiable curve.

6) suppose that  $f: G \rightarrow \mathbb{C}$  is analytic and one one ; Show that  $f'(z)$  not equal to zero for any  $z$  in  $G$ .

7) prove that an entire function has a removable singularity at infinity iff it is a constant.

## OPERATIONS RESEARCH

### Course code- MM 234

1) Solve the following LPP graphically

Maximize  $Z = 6x_1 + 5x_2$  ; subject to

$$3x_1 + x_2 \leq 160$$

$$x_1 \leq 40$$

$$x_2 \leq 130$$

$$x_1 \geq 0, x_2 \geq 0$$

2) Transform the following equation to the standard form

Minimize  $Z = -3x_1 + 4x_2 - 2x_3 + 5x_4$

Subject to  $4x_1 - 4x_2 + 2x_3 - 5x_4 = -2$

$$x_1 + x_2 + 3x_3 + 5x_4 \leq 14$$

$$-3x_1 + 4x_2 - 2x_3 + 5x_4 \geq x_1, x_2 \geq 0$$

3) Use Simplex method to solve

Maximize  $Z = x_1 + 3x_2$

Subject to  $x_1 \leq 5, x_1 + 2x_2 \leq 10, x_2 \leq 4, x_1, x_2 \geq 0$

Use Big M method to solve

4) Minimize  $Z = 6x_1 + 3x_2 + 4x_3$

Subject to  $x_1 \geq 30$

$$x_2 \leq 50, x_3 \geq 20, x_1 + x_2 + x_3 = 120, x_1, x_2, x_3 \geq 0$$

5) Use two phase method solve

Maximize  $Z=3x_1 + 4x_2+2x_3$

Subject to  $x_1 + x_2 + x_3 + x_4 \leq 30, 3x_1 + 6x_2 + x_3 - 2x_4 \leq 0, x_2 \geq 4, x_1, x_2, x_3, x_4 \geq 0,$

6) Using north west coner method solve the TP and find its optimal solution

	$M_1$	$M_2$	$M_3$	$M_4$	Supply
$W_1$	2	2	2	1	3
$W_2$	10	8	5	4	7
$W_3$	7	6	6	8	5
Demand	4	3	4	4	15

7) Find an optimal solution of the following AP

	$M_1$	$M_1$	$M_1$	$M_1$	$M_1$
$J_1$	10	11	4	2	8
$J_2$	7	11	10	14	12
$J_3$	5	6	9	12	14
$J_4$	13	15	11	10	7

8) Draw the project network find expected duration and variance of the job .and length of the project

.Also find the probability that to complete the project 3 days earlier than expected for the following

Job	predecessors	Optimistic time	Most probable time	Pessimistic time
A	---	2	5	8
B	A	6	9	12
C	A	5	14	17
D	B	5	8	11
E	C,D	3	6	9
F	--	3	12	21
G	E,F	1	4	7

8) Solve by the method of QP

Minimize  $Z= -6x_1+2x_1^2-2x_1x_2++2x_2^2$  subject to  $x_1+x_2 \leq 2, x_1, x_2 \geq 0$

9) Determine

$\max u_1^2 + u_2^2 + u_3^2$  subject to  $u_1u_2u_3 \leq 6$

10) Minimize  $u_1^2 + u_2^2 + u_3^2$  subject to  $u_1u_2u_3 \geq 10$  using forward recursion

## GRAPH THEORY (ELECTIVE)-

Course code- MM233

- 1) Determine the automorphic groups of  $P_n$  for  $n \geq 2$ .
- 2). Show that if  $G$  is a 2-regular graph, then  $\kappa(G) = \lambda(G)$ .
- 3) Prove that a tournament  $T$  is transitive iff every two vertices of  $T$  have distinct outdegrees.
- 4) Prove that a graph  $G$  has an Eulerian orientation iff  $G$  is Eulerian
- 5) Prove that a graph  $G$  contains a 1-factor iff  $\kappa(G - S) \leq |S|$  for every proper subset  $S$  of  $V(G)$ .
- 6) Prove that every  $r$ -regular bipartite graph ( $r \geq 1$ ) has a perfect matching.
- 7) Prove that for every graph  $G$   
 $\chi(G) \leq 1 + \Delta(G)$
- 8) Prove that for every graph  $G$   
 $\chi(G) \leq 1 + \max\{\delta(H)\}$   
Where the maximum is taken over all induced subgraph  $H$  of  $G$
- 9) Find the radius and diameter of the Peterson graph  $PG$ . What is the centre of  $PG$ .
- 10) Prove that for every graph  $G$  of order  $n$ .  
 $\chi(G) \geq \omega(G)$ .