

ASSIGNMENT QUESTIONS (SEMESTER 4)

FUNCTIONAL ANALYSIS

Course code: MM242

1. Show that an operator T on a Hilbert Space H is unitary iff $\{T(e_i)\}$ complete orthonormal set whenever $\{e_i\}$ is.
2. Prove that T_1 and T_2 are self adjoint operators on a Hilbert space H , prove that $T_1 T_2 + T_2 T_1$ is self adjoint
- 3) Let T be a normal operator on a finite dimensional Hilbert space H with spectrum $\lambda_1, \lambda_2, \dots, \lambda_m, \dots$. Then prove that
 - i) T is self - adjoint \Leftrightarrow each λ_i is real.
 - ii) T is positive \Leftrightarrow each $\lambda_i \geq 0$
 - iii) T is unitary $\Leftrightarrow \lambda_i = 1$ for each i .
- 4) Show that the self adjoint operator is continuous map
- 5) Prove that a Hilbert space is separable iff every orthonormal set in H is countable.
- 6) Show that an idempotent operator on a Hilbert space H is a projection on H iff it is normal.
- 7) Let x_1, x_2, \dots, x_n , be an orthonormal set in X and k_1, k_2, \dots, k_n be scalars having absolute value 1. Then $k_1 x_1 + k_2 x_2 + \dots + k_n x_n = x_1 + \dots + x_n$

Analytic Number Theory

Course code : MM244

- 1) Show that $\sum_{n=1}^{\infty} 1/p_n$ diverges
- 2) Define Mobius function, Euler's function Also find $\mu(100), \mu(12600), \varphi(100), \varphi(1200)$
- 3) Define Mangoldt function $\Lambda(n)$ Find $\Lambda(1250)\Lambda(200)$
- 4) State and prove Chinese Remainder Theorem
- 5) Find quadratic residue modulo 29, modulo 37
- 6) State and prove Gauss's Lemma
- 7) Find all primes for which 3 is a quadratic residue
- 8) Find $\left[\frac{888}{1999} \right]$
- 9) Find the primitive root of 71
- 10) Prove that 2^α has no primitive roots for $\alpha \geq 3$

COMPLEX ANALYSIS-II

Course Code: MM 241

- 1) Prove that $\prod(1+Z_n)$ converges absolutely iff $\sum|Z_n|$ converges.
- 2) Prove that a set is normal if and only if its closure is compact.
- 3) A region G_1 is conformally equivalent to G_2 if there is an analytic function $f: G_1 \rightarrow G_2$ such that f is one to one and $f(G_1) = G_2$. Prove that this is an equivalent relation
- 4) Let f be an analytic on $G = \{z: \operatorname{Re} z > 0\}$ one one with $\operatorname{Re} f(z) > 0$ for all z in G and $f(a) = a$ for some real number a show that $|f'(a)| < 1$
- 5) Prove that $\lim_{z \rightarrow 0} (\log(1+z))/z = 1$
- 6) Show that $\prod_{n=2}^{\infty} (1 - 1/n^2) = 1/2$
- 7) Let f and g be analytical functions on a region G and show that there are analytical functions f_1, g_1 , and h on G such that $f(z) = h(z) f_1(z)$ and $g(z) = h(z) g_1(z)$ for all z in G ; and f_1 and g_1 have no common zeros

Coding theory

Course code : MM243

- 1) For $n=3$ and $C = \{000, 111, 110\}$. For each word in K^3 that could be received, find the word v in the code C which IMLD will conclude was sent
- 2) Write the vector 0011 as a unique linear combination of the words $\{1000, 1100, 1110, 1111\}$
- 3) Let $H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ for the PCM for linear code C , find a) generator matrix for C^\perp b) a parity check matrix for C^\perp c) a generator matrix for C
- 4) Find a parity check matrix for $C = \langle S \rangle$, $S = \{10101, 01010, 11111, 00011, 10110\}$
- 5) Explain Golay code and find weight of C_{23}
- 6) Define Reed Mullers Codes and find $G(2, 3)$
- 7) Find generating and Parity check matrix for an extended hamming code of length 8
- 8) Find the generator polynomial of the code spanned by $S = \{010, 011, 111\}$

9) Let

$$g(x) =$$

$$1 + x^4 + x^6 + x^7 +$$

x^8 is the generator polynomial for a cyclic code with $d = 5, n = 15$.

Decode the word $w = 110011100111000$

10) If $f(x) = 1 + x + x^2$, find $[f(x)]^4$

11) Construct $GF(2^4)$ using $h(x) = 1 + x + x^4$
