M.Sc MATHEMATICS

ASSIGNMENT QUESTIONS

Semester 4

Complex Analysis II

1. Discuss the convergence of $\prod_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 0$
2. If B(E) is a closed subalgebra of C(K, C) that contains every rational function with a pole in E.
3. Let $\{u_n\}$ be a sequence of harmonic function of harmonic functions on the open disc. Show that if it converges uniformly on compact subsets of the disc, then the limit is harmonic.
4. Show that if u is harmonic then so are $u_x = \frac{\partial u}{\partial x}$ and $u_y = \frac{\partial u}{\partial y}$.
5. Show that $\frac{1}{\Gamma z}$ is an entire function of order 1
6. Let f be a meromorphic function which has only a finite number of poles in the unit disc; whenever $f(z) = 1$; whenever $z = 1$. Prove that f is a rational function.
7. Let F be a subset of a metric space (X, d) such that F^- is compact. Show that F is totally bounded.

FUNCTIONAL ANALYSIS

1.Show that an operator T on a Hilbert Space H is unitary iff T(e_i) complete orthonormal set whenever $\{e_i\}$ is.

2. Prove that $T1$ and $T2$ are self adjoint operators on a Hilbert space H, prove that $T1\,T2 + T2\,T1$ is self adjoint

3) Let T be a normal operator on a finite dimensional Hilbert space H with spectrum $\lambda_1, \lambda_2, \dots \dots, \lambda_m$, . Then prove that
i) T is self - adjoint $\Leftrightarrow$ each λi,,is real.
ii) T is positive $\Leftrightarrow$ each $\lambda i \geq 0$

iii) T is unitary $\Leftrightarrow$ λi =1 for each .

4) Show that the self adjoint operator is continuous map

5) Prove that a Hilbert space is seperable iff every ortho normal set in H is countable.

6) Show that an idem potent operator on a Hilbert space H is a projection on H
iff it is normal.
7) Let x_1 x_2 ... . . x_n, be an orthonormal set in X and k_1, k_2,. ... ... k_n be scalars
having absolute value 1. Then $k_1x_1 + k_2x_2 + \cdots ... k_nx_n = x_1 + \cdots + x_n$

CODING THEORY (SEMESTER 4)

1. Find the distance of C = { 00000 , 10011 , 11000, 11011 }.

2. Let C be the code consisting of all words of length 4 : which have been weight . Find the error patterns C detects.

3. Find the generator matrix for the cod
{ 000000 , 001011, 010101, 011110, 100110, 101101, 110011, 111000}
Find the dimension of C.

4. Find the RREF for the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

5. Find the number of different bases for each code c = < s> for
S = { 010 , 011 , 111 }
S= { 1010, 0101, 1111}
S= (0101, 1010, 1100}

6. Find an upper and lower bound for the maximum number of code words in linear code of
   i) Length n = 15 and distance d = 5
   ii) Lengh n = 15 and distance d = 3 .

7. Find two generator of degree 4 for a linear cyclic code of length 7.

ANALYTIC NUMBER THEORY

1. Prove that (a , b) = (a + b , (a , b))
2. Prove that if $2^n + 1$ is prime , then n is a power of 2.
3. If (a , B) = 1 and ab = c^n , Prove that a = x^n , b = y^n for some x and y .
4. Prove that $\sum_{\frac{d^2}{n}} \mu(d) = \mu^2(n)$
5. Prove that $\mu(x, n) = \sum_{\frac{d}{n}} \mu(d)(\frac{x}{d})$
6. Prove that $\sigma_1(n) = \sum_{\frac{d}{n}} \emptyset(d)\sigma_0(\frac{d}{n})$
7. Solve each of the following
   i) 25x = 15 (mod29)
   ii) 5x = 2 (mod 26)
8. The prime p = 71 has 7 as a primitive root. Find all primitive roots 71 and also find a primitive roots of p^2 and $2p^2$.
9. Prove that $n^4 + 1$ is composite if n > 1.