SECOND SEMESTER M.Sc MATHEMATICS

ASSIGNMENT QUESTIONS (2022 ADMISSION)

ALGEBRA

Questions

1. Show that set of all 2 X 2 matrices with real numbers as entries and determinant 1 is a group under matrix multiplication. Is it an abelian group?
2. Law of exponents for abelian group states that if ɑ and b are any tow elements of an abelian group and n any integer, then $(ab)^n = a^n b^n$. Is it true for a non-abelian group?
3. Find the order of the group U(12). Find the order of all elements in U(12).
4. Show that U(14) is cyclic
5. Find an example of an abelian group which is not cyclic
6. How many generators are there for a cyclic group of order 10.
7. Prove that Sn is non-abelian for $n > 2$.
8. Is Z under addition isomorphic to Q under addition?
9. Find an isomorphism from the group of integers under addition to the group of even integers under addition
10. Let n be an integer greater than 1. Let $H = \{0, \pm n, \pm 2n, \pm 3n, \ldots..\}$. Find all cosets of H in Z.

REAL ANALYSIS – II

1. Show that there exist uncountable sets of zero measure.
2. Show that monotone functions are measurable.
3. Let $f(x) = x\, sin\left(\frac{1}{x}\right)\ if x \neq 0\ and\ 0\ if\ x = 0.$ Find the four dérivâtes at $x = 0$
4. Describe the ring generated by the finite open intervals.
5. Show that every algebra is a ring and every σ algebra is a σ ring but the converse is not true.
6. Prove that the limit of pointwise convergent sequence of measurable function is measurable.

COMPUTER PROGRAMMING – C++

1. A cricket team has the following table of batting figures for a series of test matches.

Player's name	Runs	Innings	Times not out
Sachin	8430	230	18
Saurav	4200	130	9
Rahul	3350	105	11
.	.	.	.
.	.	.	.

Write a program to read the figures set out in the above form, to calculate the batting average and to print out the complete table including the averages.

2. Write a program to evaluate the following functions to 0.0001% accuracy.
   (a) Sin x= x- $x^3/3! + x^5/5! - x^7/7! + \ldots\ldots$
   (b) SUM= $1 + (1/2)^2 + (1/3)^3 + (1/4)^4 + \ldots.$
   (c) Cos x= $1 - x^2/2! + x^4/4! - x^6/6! +, \ldots.$

3. Write a program to print a table of values of the function y= e^{-x} for x varying from 0 to 10 in steps of 0.1. The table should appear as follows.

   Table for Y= EXP [-X]

X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0									
1.0									
.									
.									
.									
9.0									

4. Write a program to calculate the variance and standard deviation of N numbers.

$$\text{Variance} = \frac{1}{N}\sum_{i=1}^{N}(x1 - \bar{x})^2$$

$$\text{Standard deviation} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$\text{Where } \bar{x} = \frac{1}{N}\sum_{i=1}^{N} X1$$

5. An electricity board charges the following rates to domestic users to discourage large consumption of energy:

   For the first 100 units- 60P per unit
   For the next 200 units- 80P per unit
   Beyond 300 units- 90P per unit

All users are charged a minimum of Rs. 50.00. of the total amount is more than Rs.300.00 then an additional surcharge of 15% is added.

Write a program to read the names of users and number of units consumed and print out the charges with the names.

TOPOLOGY –II

QUESTIONS

1. Prove that every open continuous image of a locally compact space is locally compact
2. Prove that every closed subspace of a locally compact space is locally compact.
3. Prove that an infinite product of discrete space may not be discrete.
4. Prove that a topological space (X,τ) is a Hausdorff space iff every net in X can converge to atmost one point.
5. Show that every filter $\mathcal{F}$ on X is the intersection of all the ultrafilters finer than $\mathcal{F}$
6. Show that σ^k is the smallest convex set which contains all vertices of σ^k