

**M.Sc mathematics (2022 Admission)**  
Assignment questions ( semester 4)

**Complex Analysis II**

1. Discuss the convergence of  $\prod_{n=1}^{\infty} \frac{1}{n^p}$  for  $p > 0$
2. If  $B(E)$  is a closed subalgebra of  $C(K, \mathbb{C})$  that contains every rational function with a pole in  $E$ .
3. Let  $\{u_n\}$  be a sequence of harmonic function of harmonic functions on the open disc. Show that if it converges uniformly on compact subsets of the disc, then the limit is harmonic.
4. Show that if  $u$  is harmonic then so are  $u_x = \frac{\partial u}{\partial x}$  and  $u_y = \frac{\partial u}{\partial y}$ .
5. Show that  $\frac{1}{\Gamma z}$  is an entire function of order 1
6. Let  $f$  be a meromorphic function which has only a finite number of poles in the unit disc; whenever  $f(z) = 1$ ; whenever  $z = 1$ . Prove that  $f$  is a rational function.
7. Let  $F$  be a subset of a metric space  $(X, d)$  such that  $F^-$  is compact. Show that  $F$  is totally bounded.

## FUNCTIONAL ANALYSIS(SEMESTER 4) ASSIGNMENT QUESTIONS

1. Show that an operator  $T$  on a Hilbert Space  $H$  is unitary iff  $T(e_i)$  complete orthonormal set whenever  $\{e_i\}$  is.
2. Prove that  $T_1$  and  $T_2$  are self adjoint operators on a Hilbert space  $H$ , prove that  $T_1 T_2 + T_2 T_1$  is self adjoint
- 3) Let  $T$  be a normal operator on a finite dimensional Hilbert space  $H$  with spectrum  $\lambda_1, \lambda_2, \dots, \lambda_m, \dots$ . Then prove that
  - i)  $T$  is self - adjoint  $\Leftrightarrow$  each  $\lambda_i$  is real.
  - ii)  $T$  is positive  $\Leftrightarrow$  each  $\lambda_i \geq 0$
  - iii)  $T$  is unitary  $\Leftrightarrow \lambda_i = 1$  for each  $i$ .
- 4) Show that the self adjoint operator is continuous map
- 5) Prove that a Hilbert space is separable iff every orthonormal set in  $H$  is countable.
- 6) Show that an idempotent operator on a Hilbert space  $H$  is a projection on  $H$  iff it is normal.
- 7) Let  $\{x_1, x_2, \dots, x_n\}$ , be an orthonormal set in  $X$  and  $\{k_1, k_2, \dots, k_n\}$  be scalars having absolute value 1. Then  $k_1 x_1 + k_2 x_2 + \dots + k_n x_n = x_1 + x_2 + \dots + x_n$

**CODING THEORY ( SEMESTER 4 )**  
**ASSIGNMENT QUESTIONS**

1. Find the distance of  $C = \{ 00000, 10011, 11000, 11011 \}$ .
2. Let  $C$  be the code consisting of all words of length 4 : which have even weight . Find the error patterns  $C$  detects.
3. Find the generator matrix for the code  
 $\{ 000000, 001011, 010101, 011110, 100110, 101101, 110011, 111000 \}$   
Find the dimension of  $C$ .
4. Find the RREF for the matrix
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$
5. Find the number of different bases for each code  $C = \langle S \rangle$  for  
 $S = \{ 010, 011, 111 \}$   
 $S = \{ 1010, 0101, 1111 \}$   
 $S = \{ 0101, 1010, 1100 \}$
6. Find an upper and lower bound for the maximum number of code words in linear code of  
i) Length  $n = 15$  and distance  $d = 5$   
ii) Length  $n = 15$  and distance  $d = 3$  .
7. Find two generators of degree 4 for a linear cyclic code of length 7.

ANALYTIC NUMBER THEORY ( SEMESTER 4 )  
 ASSIGNMENT QUESTIONS

1. Prove that  $(a, b) = (a + b, (a, b))$
2. Prove that if  $2^n + 1$  is prime, then  $n$  is a power of 2.
3. If  $(a, B) = 1$  and  $ab = c^n$ , Prove that  $a = x^n$ ,  $b = y^n$  for some  $x$  and  $y$ .
4. Prove that  $\sum_{\substack{d|n}} \mu(d) = \mu^2(n)$
5. Prove that  $\mu(x, n) = \sum_{\substack{d|n}} \mu(d) \left(\frac{x}{d}\right)$
6. Prove that  $\sigma_1(n) = \sum_{\substack{d|n}} \phi(d) \sigma_0\left(\frac{n}{d}\right)$
7. Solve each of the following
  - i)  $25x \equiv 15 \pmod{29}$
  - ii)  $5x \equiv 2 \pmod{26}$
8. The prime  $p = 71$  has 7 as a primitive root. Find all primitive roots 71 and also find a primitive roots of  $p^2$  and  $2p^2$ .
9. Prove that  $n^4 + 1$  is composite if  $n > 1$ .