B.Sc. MATHEMATICS FIFTH SEMESTER

ASSIGNMENT QUESTIONS

ABSRACT ALGEBRA I **COURSE CODE –MM1545**

QUESTIONS

1. Draw the Lattice diagram of z_{32}.
2. Find the number of generators of cyclic groups of order 12 , 18, 30
3. Show that (Q , +) is not a cyclic group.
4. Let Q be the set of rationals and let $Q(\sqrt{2}) = \{a + b\sqrt{2} \ : a, b \in Q\}$. Show that $Q(\sqrt{2})$ is a group under addition.
5. Let G be the set of all 2x2 matrices $\begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}$ where $a \neq 0$. Prove that G is abelian group under the matrix multiplication.
6. If G is a group of even order , Prove that it has an element $a \neq e$. Prove that G is an abelian group under matrix multiplication.
7. In$(Z,+)$, let $H = set\ of\ all\ multiples\ of$ 3 and $K = set\ of\ all\ multiples\ of$ 5.
   Show that H and$K$ are subgroups of Z . Also describe $H \cap K$
8. For each binary operation $*$ defined below say whether the following is a group or not
   a) Define $*$ $on\ Z$ by $a * b = a - b$
   b) Define $*$ $on\ Z$ by $a * b = ab$
   c) Define $*$ $on\ R^+$by $a * b = ab$
   d) Define $*$ $on\ Q$ by $a * b = ab$
9. Express the following as the product of disjoint cycles
   a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 7 & 1 & 6 & 4 \end{pmatrix}$
   b)$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 9 & 5 & 4 & 7 & 8 & 1 & 6 & 10 & 2 \end{pmatrix}$
   c)$(1\ 3\ 2\ 5)(1\ 4\ 3)(2\ 5\ 1)$
   d)$(1\ 4\ 3\ 2)(2\ 4\ 1)$

5. Compute $a^{-1}ba$where $a = (1\ 3\ 4)$ $and\ b = (2\ 3\ 5\ 4)$

6. If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$ $and\ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$ find $\alpha\beta\ and\ \alpha^{-1}$,α^2.

REAL ANALYSIS – I COURSE CODE – 1541

QUESTIONS

1. Find all real numbers x such that
   a) $x^2 > 3x + 4$
   b) $1 < x^2 < 4$
2. Prove that $\frac{c_1+c_2+\cdots+c_n}{\sqrt{n}} \leq ({c_1}^2 + {c_2}^2 + \cdots + {c_n}^2)^{\frac{1}{2}} \leq c_1 + c_2 + \cdots + c_n$
3. Prove that $\sqrt{3}$ is not a rational number
4. Prove the sequence $\{n\}$ is divergent.
5. Prove the limit of $x_n = \frac{1}{2}[x_{n-1} + x_n]$
6. Show that $x_{n+1} = \frac{1}{2+x_n}$ is contractive and find its limits.
7. Determine whether the following limits exists and justify your answer
   a) $\lim_{x\to 0} cos\frac{1}{x}$ b) $\lim_{x\to 0} x\,sin\frac{1}{x}$ c) $\lim_{x\to 0} x\,cos\frac{1}{x}$ d) $\lim_{x\to 0} x\,sin\frac{1}{x^2}$

8. Find the following

a). $\lim_{x\to 2}\frac{x^2-4}{x-2}$

b) $\lim_{x\to 1}\frac{\sqrt{x-1}}{x-1}$

c) $lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt{1+3x}}{2+2x^2}$ $x > 0$

DIFFERENTIAL EQUATIONS MM -1543

ASSIGNMENT QUESTIONS

1. Solve the equations by the method of Integrating factors.
   (a) $\frac{dy}{dx} + 3y = e^{-2x}$
2. Solve the equation using variable separable method.
   (a) $\frac{dy}{dt} - 2y = 0$

3.Show that the equations are exact and solve

$$2xydx + x^2dy = 0$$

4.Find the integrating factor and solve the following equations:

$$(y-x^2)dx+(x^2\sin y-x)dy=0$$

5. Find a general solution by the method of variation of parameters.

$$y''+9y=\sec\ 3x$$

6 Find a general solution of the differential equations given below:

(a) $y''+4y=\sin\ 3x$

7.Find the general solution of the equation:

$$x^2y''-7xy'++12y=0$$

8.Solve the initial value problem:

$$x^2y''+xy'+9y=0;\ y(1)\ =2, y'(1)=0$$

9.Find the general solution of the equation:

$$x^2y''-9xy'+25y=0$$

VECTOR ANALYSIS **course code : MM - 1544**

ASSIGNMENT QUESTIONS

1.If $f(x,\ y,\ z)$ =$3X^2\,Y-\ Y^3Z^2$ find ∇f at the point (1, −2,-1).

2. Find the unit vector normal to the surface $x^2+2y^2+z^2\ =7$ at (1, -1,2).

3. Compute the divergence and curl of the vector point functions.

a). $F^-\ =\ X^2YZi-\ 2X\ Z^3j+X\ Z^2k.$

b). $F^-\ =x^2i-2xyj+3x^2\ zk$

4. Show that the divergence of inverse square field is zero.

5. Evaluate $\int_c\ 2xydx+(x^2+\ y^2)\ dy$ along the circular arc c given by $x=cost, y=\sin t\quad (0\leq t\leq\frac{\pi}{2})$

6. Evaluate $\int_C\ F.dr$,where $F=X^2-Y^2i+xyj$ and curve C is the arc of the curve $y=X^3$from (0,0) to (2,8).

7. Find the total work done in moving a particle in a force field given by $F^{-} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ $from\, t = 1\ to\ t = 2$

10. Evaluate using Green's Theorem $\oint 3xydx + 2xydy$, where C is the rectangle bounded by $x = -2$, $x = 4$, $y = 1$ and $y = 2$.

11. Verify Stoke's theorem when $F = x^2 i + y^2 j + z^2 k$,S is the upper hemisphere $z = \sqrt{a^2 - x^2 - y^2}$

COMPLEX ANALYSIS

COURSE CODE: MM-1542

QUESTIONS

1. Find the square root of $-5 - 12i$
2. Using the Cauchy –Riemann equations verify the following is analytic or not
   i) $x^2 - y^2 + 2ixy$
   ii) $x^2 + y^2 - 2ixy$
3. Find the solutions of $e^z = i$
4. Show that $\sin iy = i\, sinhy$
5. Evaluate $\int_c f\ \ where\, f(z) = x^2 + i\, y^2$, where c is given by $z(t) = t^2 + it^2$ $, 0 \le t \le 1$
6. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{z^n}{n!}$