M.Sc. Mathematics

Semester 3

Assignment questions

Complex Analysis I

course code: MM231

1. Derive the following sets
   i) $\{z : e^z = i\}$
   ii) $\{z : e^z = -i\}$
   iii) $\{z : e^z = -1\}$
   iv) $\{z : cosz = 0\}$
2. $Calculate\ the\ following$
   i) $\int_0^{\pi} \frac{cos2\theta d\theta}{1-2acos\theta + a^2} a^2 \leq 1$
   ii) $\int_0^{\pi} \frac{d\theta}{(a+cos\theta)^2}\ where\ a > 1$
3. Verify the following
   i) $\int_0^{\infty} \frac{cosaxdx}{(1+x^2)^2} = \frac{\pi(a+1)e^{-a}}{4}$ if a > 0
   ii) $\int_0^{\infty} \frac{logxdx}{(1+x^2)^2} = \frac{-\pi}{4}$
   iii) $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$
   iv) $\int_0^{2\pi} logsec^2 2\theta d\theta = 4\int_0^{\pi} logsin\theta d\theta$
4. Evaluate the following

i) $\int_{\gamma} \frac{e^z - e^{-z}\ dz}{z^n}$ where n is a positive integer and $\gamma(t) = e^{it}\quad 0 \leq t \leq 2\pi$

ii) $\int_{\gamma} \frac{z^2+1\ dz}{z(z^2+4)} where\ \gamma(t) = re^{it}\quad 0 \leq t \leq 2\pi$

iii) $\int_{\gamma} \frac{dz}{z^2+1}\quad where\ \gamma(t) = 2e^{it}, 0 \leq t \leq 2\pi$

5. Evaluate
   i) $\int_{\gamma} \frac{e^{iz}dz}{z^2}\quad \gamma(t) = e^{it}, 0 \leq t \leq 2\pi$
   ii) $\int_{\gamma} \frac{sinzdz}{z^3}\quad \gamma(t) = e^{it}\quad 0 \leq t \leq 2\pi$
   iii) $\int_{\gamma} \frac{logz}{z^n} dz\quad \gamma(t) = 1 + \frac{1}{2}e^{it}, 0 \leq t \leq 2\pi$

Operation Research

course code: MM234

1. Use simplex method to solve Maximize $z = x_1 + 3x_2$ Subject to $x_1 \leq 5$;

$x_1 + 2x_2 \leq 10$; $x_2 \leq 4$,

$$x_1, x_2 \geq 0$$

2. Use big M method solve.

Minimize $Z = 2y_1 + 4y_2$

$$Subject\ to\ 2y_1 - 3y_2 \geq 2,$$

$$-y_1 + y_2 \geq 3; \quad y_1, y_2 \geq 0$$

3. Find the optimum basic feasible solution to the following

	D_1	D_2	D_3	D_4	Available
O_1	7	9	3	2	16
O_2	4	4	3	5	14
O_3	6	4	5	8	20
Requirement	11	9	22	8	50

4)Find the optimal assignment of the following

	J_1	J_2	J_3	J_4
m_1	10	9	7	8
m_2	5	8	7	7
m_3	5	4	6	5
m_4	2	3	4	5

5) Consider a project consisting of nine jobs (A, B, C,….,I) with the following precedence relations and time estimates.

Job	Predecessor	Time (Days)
A	--	15
B	--	10
C	A,B	10
D	AB	10
E	B	5
F	D,E	5

G	C ,F	20
H	D ,E	10
I	G ,H	15

a. Draw the project network for this problem designating the jobs by arcs and event by nodes. b. Determine the earliest completion time of the project, and identify the critical path.

c. Determine a project schedule listing the earliest and latest starting times of each job. Also identify the critical job.

6). Minimize $f = (x_1 - 2)^2 + x_2^2$ subject to x_1^2 - $x_2 - 1 \le 0$, $x_1 \ge 0, x_2 \ge 0$, Obtain the solution graphically.

7)Minimize ${u_1}^2 + {u_2}^2 + {u_3}^2$ $subject\ to$ $u_1 + u_2 + u_3 \ge 0$, $u_1, u_2, u_3 \ge 0$

GRAPH THEORY

course code: MM233

ASSIGNMENT QUESTIONS

1. Prove that if G is a cubic graph , then $K(G) = \lambda(G)$
2. Prove that for every tree $T_m$of order m ≥ 2 and every integer$n \ge$ 2, $r(T_m, K_m) = (m-1)(n-1) + 1$
3. Prove that a graph G is Eulerian if and only if every vertex is of even degree.
4. Prove that every tournament contains a Hamiltonian path.
5. Determine the chromatic number of each of the following:
   a) The Petersen graph
   b) The n – cube Q_n.
   c) The wheel $W_n \cong C_n + K_1$

FUNCTIONAL ANALYSIS

course code: MM232

1. Let ‖ ‖ and ‖ ‖' be two norms on a linear space X. Then the norm ‖ ‖ is equivalent to the norm ‖ ‖' if and only if there exists $\alpha, \beta > 0$ such that β‖ ‖' ≤‖x‖≤ α‖ ‖ for all x ϵ X.
2. Let l^2 and letl_n be defined as $l_n(k) = \begin{Bmatrix} 1 & if\ k = n \\ 0 & otherwise \end{Bmatrix}$

   Show that $l_1, l_2, l_3, l_4, \ldots$...is compact.

3. Show by an example that an infinite dimensional subspace of a normal space X may not be closed in X.
4. Let X $\neq$ 0 and y be normed space. Prove that BL(X,Y) is Banachiff Y Banach.
5. Show that any two norms on a finite dimensional linear space are equivalent.
6. Show that a Banach space cannot have a countably infinite basis.

7. Let X be a normed linear space and Y a closed subspace of X with Y $\neq$ X ,if $0 < r < 1$ prove that there exist $X_r \epsilon X \; such \; that$ $\|X_r\| = 1$And $r < d(X_r , Y) \leq 1$
8. Prove that

   i) $d(ax , ay) = |a|$d(x,y)

   ii)$d(a + x , a + y) = d(x , y)$

   where d is a metric induced by on a normed space X
9. For $x \in C_\infty$, let $f(x) = \sum_{n=1}^{\infty} x(n)$ Show that f is not continuous.
10. Find the norm of the linear functional f defined by $f(x) = \int_{-1}^{0} x(t)dt - \int_{0}^{1} x(t)dt$ where$x \in [-1 , 1]$
11. Show that a closed subspace of a Banach Space is Banach.
12. Show that the inverse of $F : X \rightarrow Y$ is continuous bijective linear map.