M.Sc. MATHEMATICS

FIRST SEMESTER

ASSIGNMENT QUESTIONS (2024 ADMISSION)

LINEAR ALGEBRA (Course Code: MM211)

QUESTIONS

1. If V is finite dimensional, prove that any linear map on a subspace can be extended be a linear map on V.

2. Prove that the real vectorspace consisting of all continuous real valued functions on the interval [0,1] is infinite dimensional.

3. Prove that if there exists a linear map on V whose nullspace and range are both finite dimensional then V is finite dimensional.

4. Prove that if V is finite dimensional with dim V >1 then the set of noninvertible operation on V is not a subspace of L(V).

5. Suppose P $\in$ P(z) has degree m . Prove that p has m distinct roots iff p and its derivative p ' have no common roots.

6. Find the Eigen values of T$\in$ L(F) defined by T (w, z) = (-z ,w) when (i) F= R (ii) F = C.

Differential Equations (course code: MM213)

1. Find the general solution of the equation $y'' - 2y' + 5y = 25x^2 + 12$.
2. Find a function on $-1 \leq x \leq 1, 0 \leq y \leq 1$ which does not satisfy a Lipschitz condition.
3. Consider the differential equation $y' = 2xy$ and find a power series expansion$\sum a_n\, x^n$.
4. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of the power series in x.
5. In the differential equation $x^3(x-1)y'' - 2x(x-1)y' + 3xy = 0$ locate and classify the singular points on the x axis.

6. Determine the nature of the point x=0 for the differential equation $xy'' + (sinx)y = 0$
7. Verify that $sin^{-1}(x) = xF(\frac{1}{2},\frac{1}{2},\frac{3}{2},x^2)$.
8. Transform the Chebyshev's equation $(1-x^2)y'' - xy' + p^2y = 0$ into a hypergeometric equation by replacing x by $t = \frac{1}{2}(1-x)$ and show that its general solution near x=1 is $y = c_1F(p,-p,\frac{1}{2},\frac{1-x}{2})$.
9. Determine the nature of the point $x = \infty$ for Legendre's equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$

10. Show that

    i) $\frac{d}{dx}[J_0(x)] = -J_1(x)$

    ii) $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$

REAL ANALYSIS – I (Course code: MM212)

Questions.

1. Determine which of the following functions are bounded variation on [0,1]

   a) $f(x) = xsin\left(\frac{1}{x}\right)\ if x \neq 0\ and\ 0\ if\ x = 0$

   b) $f(x) = \sqrt{x}\sin x, if x \neq 0\ and\ 0\ if\ x = 0$

2. Give an example of a function which is not Riemann integrable but Stieljesintegrable.

3. If $f_n \to f$ uniformly and f_n is bounded on a set S. prove that $\{f_n\}$ is uniformly bounded on S.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} x+y & if\ x \neq y \\ 1 & if\ x = y \end{cases}$. Prove that

   $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exists.

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} x^2+y^2 & if\ both\ x\ and\ y\ are\ rationals \\ 0 & Otherwise \end{cases}.$$

   Determine the points of $\mathbb{R}^2$ which $f_x\ and\ f_y$ exists.

TOPOLOGY – I (course code: MM214)

Questions.

1. Show that the set C of all complex numbers is a metric space with respect to the metric d, defined by

   $d(z_1, z_2) = \frac{|z_1 - z_2|}{[(1+|z_1|^2)(1+|z_2|^2]^{\frac{1}{2}}}$ for all z_1, z_2 in C.
2. Prove that A metric subspace (Y, d) of a complete metric space (X, d) is complete iff Y is closed.
3. Let E be a totally bounded subset of a metric space X. Show that every sequence { a_n} in E contains a Cauchy subsequence.
4. Let T be the class of subsets of N consisting of ∅ and all subsets of N of the form E_n ={ n , n +1 , n+2 , ….. } with $n \in N$.

   i) Show that T is a Topology on N

   ii) List the open sets containing the positive integer 6
5. Prove that a Topological space is compact iff every family of closed sets 7with empty intersection has a finite subfamily with empty intersection
6. Prove that every infinite subset of the Topological space has a limit point.
7. Prove that every compact Hausdorff space is a T_4 –space.