

**M.Sc. MATHEMATICS**  
**SECOND SEMESTER**  
**ASSIGNMENT QUESTIONS (2024 ADMISSION)**

**ALGEBRA**

**Course Code : MM221**

**Questions**

1. Show that set of all  $2 \times 2$  matrices with real numbers as entries and determinant 1 is a group under matrix multiplication. Is it an abelian group?
2. Law of exponents for abelian group states that if  $a$  and  $b$  are any two elements of an abelian group and  $n$  any integer, then  $(ab)^n = a^n b^n$ . Is it true for a non-abelian group?
3. Find the order of the group  $U(12)$ . Find the order of all elements in  $U(12)$ .
4. Show that  $U(14)$  is cyclic
5. Find an example of an abelian group which is not cyclic
6. How many generators are there for a cyclic group of order 10.
7. Prove that  $S_n$  is non-abelian for  $n > 2$ .
8. Is  $\mathbb{Z}$  under addition isomorphic to  $\mathbb{Q}$  under addition?
9. Find an isomorphism from the group of integers under addition to the group of even integers under addition
10. Let  $n$  be an integer greater than 1. Let  $H = \{0, \pm n, \pm 2n, \pm 3n, \dots\}$ . Find all cosets of  $H$  in  $\mathbb{Z}$ .

**REAL ANALYSIS – II****Course Code : MM222**

1. Show that there exist uncountable sets of zero measure.
2. Show that monotone functions are measurable.
3. Let  $f(x) = x \sin\left(\frac{1}{x}\right)$  if  $x \neq 0$  and 0 if  $x = 0$ . Find the four dérivâtes at  $x = 0$
4. Describe the ring generated by the finite open intervals.
5. Show that every algebra is a ring and every  $\sigma$  algebra is a  $\sigma$  ring but the converse is not true.
6. Prove that the limit of pointwise convergent sequence of measurable function is measurable.

**COMPUTER PROGRAMMING – C++****Course code : MM224**

1. A cricket team has the following table of batting figures for a series of test matches.

Player's name	Runs	Innings	Times not out
Sachin	8430	230	18
Saurav	4200	130	9
Rahul	3350	105	11
.	.	.	.
.	.	.	.

Write a program to read the figures set out in the above form, to calculate the batting average and to print out the complete table including the averages.

2. Write a program to evaluate the following functions to 0.0001% accuracy.

(a)  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

(b)  $\text{SUM} = 1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{4}\right)^4 + \dots$

(c)  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

3. Write a program to print a table of values of the function  $y = e^{-x}$  for  $x$  varying from 0 to 10 in steps of 0.1. The table should appear as follows.

Table for  $Y = \text{EXP} [-X]$

X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0									
1.0									
.									
.									
.									
9.0									

4. Write a program to calculate the variance and standard deviation of  $N$  numbers.

$$\text{Variance} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2$$

$$\text{Standard deviation} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

$$\text{Where } \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

5. An electricity board charges the following rates to domestic users to discourage large consumption of energy:

For the first 100 units- 60P per unit

For the next 200 units- 80P per unit

Beyond 300 units- 90P per unit

All users are charged a minimum of Rs. 50.00. If the total amount is more than Rs. 300.00 then an additional surcharge of 15% is added.

Write a program to read the names of users and number of units consumed and print out the charges with the names.

**QUESTIONS**

1. Prove that every open continuous image of a locally compact space is locally compact
2. Prove that every closed subspace of a locally compact space is locally compact.
3. Prove that an infinite product of discrete space may not be discrete.
4. Prove that a topological space  $(X, \tau)$  is a Hausdorff space iff every net in  $X$  can converge to atmost one point.
5. Show that every filter  $\mathcal{F}$  on  $X$  is the intersection of all the ultrafilters finer than  $\mathcal{F}$
6. Show that  $\sigma^k$  is the smallest convex set which contains all vertices of  $\sigma^k$