

Mathematics

1. Find the $\frac{dy}{dx}$ if $y = x^{\sin x} + (\sin x)^x$
2. Use Leibnitz's Theorem, if $y = \cos(m \sin^{-1} x)$
PT $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$
3. Use Rolles Theorem, verify for $f(x) = x^3 - 9x$ in the interval $[0,3]$
4. Examine the maxima and minima of the function $z = x^3 + 3xy^2 + 15x^2 - 15y^2 + 72x$
5. Solve the differential equation by the method of undermined coefficients
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12e^x$$
6. By eliminating the arbitrary function, Obtain the p.d.e. $z = f(x+ct) + g(x-ct)$
7. Solve $z^2(p^2x^2 + q^2) = 1$
8. Solve $(w+y+z)\frac{\partial w}{\partial x} + (w+x+z)\frac{\partial w}{\partial y} + (w+x+y)\frac{\partial w}{\partial z} = x+y+z$
9. Find the inverse Laplace transform of $\frac{s^2+6}{(s^2+1)(s^2+4)}$
10. Using convolution theorem evaluate the following
$$L^{-1}\left[\frac{S}{(s^2+1)(s^2+4)}\right]$$
11. Find the Laplace transform of $t \sin 3t \cos 2t$
12. Find the greatest common divisor of 58 and 86 and express it as the linear combination of these two integers.
13. If n is any natural number, prove that $n(n+1)(n+2)$ is divisible by 6.
14. If $(a,b) = 1$ and $(a,c) = 1$ show that $(a,bc) = 1$
15. Find the remainder when 4^{17} is divisible by 13
16. Show that $36! + 1037 = m(1517)$
17. Show that 4th power of any number is of the form $13m$ or $13m+1$
18. If $\tan \frac{\theta}{2} = \tanh \frac{u}{2}$ show that $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$
19. Separate into real and imaginary parts of the expression $\sin(x+iy)$
20. Find the bilinear transformation which maps the points $0,1,-1$ into the points $(i, \infty, 0)$
21. Show that the transformation $w = \frac{1}{z}$ transforms the circle $x^2 + y^2 - 6x = 0$ into a straight line into the w plane.