School of Distance Education (SDE), University of Kerala

M.Sc Mathematics (Semester I)

Assignment Topics

MM212 - Real Analysis

1. If $f(x) = x^2\ Cos\left(\frac{1}{x}\right), x \neq 0$ & $f(0) = 0$ Prove that f is bounded variation. Show by an example by that the boundedness of $f^{'}$ is not necessary for f to be of bounded variation.

2. If *f* and g are complex valued functions defined by $f(t) = e^{2\pi it}$ if $t \in\ [0,1]$ and $(t) = e^{2\pi it}\ t \in\ [0,2]$, Prove that *f* and g have same graph but not equivalent.

3. Let α be a continues function of bounded variation on $[a,b]$. Assume $g \in R(\alpha)$ on$[a,b]$ and define$\beta(x) = \int_a^x g(t)d\alpha(t)$, if $x \in [a,b]$. Show that if *f* is ↑ on $[a,b]$ there exists a point x_0 in $[a,b]$ such that $\int_a^b f d\beta\ = f(a)\ \int_a^{x_0} g d\alpha + f(b)\ \int_{x_0}^b g d\alpha$

4. Prove that $\sum_{n=1}^{\propto} \frac{x}{n^{\alpha}(1+nx^2)}$ converges uniformly on every finite interval in R if$\alpha > \frac{1}{2}$. Is the convergence uniform on R?

5. Check whether the functions are uniformly continuous or not?

$$f: R^2\ \to R\ defined\ by$$

$$f(x,y) = \begin{Bmatrix} 1\ if\ (x,y) = (0,0) \\ Sin\ (x,y)\ \ if\ (x,y) \neq (0,0) \end{Bmatrix}$$

6. Let $f: R^2\ \to R\ defined\ by$

$f(x,y) = \begin{Bmatrix} 0\ if\ (x,y) = (0,0) \\ 1\quad if\ (x,y) \neq (0,0) \end{Bmatrix}$ Show that *f* is not continuous at (0,0). Although both partial derivatives of *f* exists at (0,0).

7. $f: R^2 \rightarrow R$ $defined\ by$

$f(x,y) = x^2 + y^2$ if x and y are both rational and $f(x,y)$ =0 otherwise. Determine the points of R^2 at which (i) f_x exists (ii) f_y exists.

8. If $f \in R(\alpha)$ on $[a,b]$ and if $\int_a^b f d\alpha = 0$ for every f which monotonic on $[a,b]$. Prove α must be a constant.

9. Use Euler's Summation formula or integration by parts is a Rimann Sum is integral. Show that must be a constant.

9. Use Euler's Summation formula or integration by parts is a Rimann Sum is integral. Show that $\sum_{k=1}^{n} \frac{1}{x} = logn - \int_1^n \frac{x-[x]}{x^2}\, dx + 1$

10. Show that a sequence of functions for which $\lim_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 \lim_{n\to\infty} f_n(x)dx$

MM-213 Paper III- Differential Equations

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1. Explain the method of undetermined coefficients for finding a particular integral of the equation $y'' + py' + qy = e^{ax}$, where *p, q* and *a* are constants.

2. Using Picard's method to find y_1, y_2 *and* y_3 of $y' = y^2$, $y_{(0)} = 1$ and compare with the exact solution.

3. Solve by variation of parameters.

$(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2$ **.**

4. Find the general solution of the Gauss' hyper geometric equation.

$x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0$ **near the singular point $x = 0$.**

5. Consider the equation $y'' = xy' + y = 0$. Find its general solution is the form $y = a_0y_1 + a_1y_2(x)$ where $y_1(x)$ and $y_2(x)$ are power series.

6. Find the general solution of $(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0$ at $x = 3$.

7. Obtain the orthogonal property to Legendre polynomials.

8. Find the relation between Bessel functions and Trigonometric functions.

9. Show that $\frac{2p}{x}.J_p(x) = J_{p-1}(x) + J_{p+1}(x)$.

10. Find the general integral of $yzp + xzq = xy$.

11. By Jacobis method, solve the equation:

$z^2 + zu_x - u_x^2 - u_y^2 = 0$.

12. Show that the equations:

f=xp-yq-x=0 and g= $x^2p + q - xz = 0$ **are compatible and find one – parameter family of common solutions.**

Reduce the equation.

13. $4u_{xx} - 4u_{xy} - 5u_{yy} = 0$ **to a canonical form.**

14. Solve the equation:

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{e^2}\frac{\partial^2 y}{\partial t^2}$; $0 < x < \infty, \ \ t > 0$

The initial conditions are

$Y(x, 0) = u(x);\ y_1(x, 0) = v(x), x \geq 0$.

The boundary conditions is $Y(0, t) = 0, t \geq 0$.

15. State the following Boundary value problems

i. The Dirichiet problem

ii. The Neumann problem

iii. The Robin problem.

MM214- Topology I

1. If (X, d) is a metric space. Then prove that there is a bounded metric on X which is equivalent to d.

2 Suppose X is a metric space, A and B are sub sets of X. Then

a) Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$

b) Prove that $\overline{A \cap B} \subset \bar{A} \cap \bar{B}$

c) Find an example to show that we do not generally get equality in part (b)

3. Describe the boundaries of the following subsets of R.

a) Q b) $R/_{Q}$ c) [0,1] d) $(0,1)$ e) **Z**

4 . Let $(x_n: n \in W)$ is a convergent sequence in a metric space (X, d). Then prove that it is a Cauchy sequence. Is the converse true? Justify your answer.

5. Prove that any open interval (a, b) is homeomorphic to $(0,1)$.

6. If $X_1 \times X_2$ is a nonempty product space,then prove that the projections π_1 and π_2 are both open,continous and onto functions but need not be closed.

7. Prove that the Topologies Sine Curve is connected but not pathwise connected.

8. Prove that if f:X→Y, then f is continous if nd only if f^{-1}(B) is open in X for each member B of some base for Y.Prove that the same result with "base "replaced by "sub base".

9. Suppose X is an infinite set with co finite topology .Prove that X is T_1 but not T_2.

10. Prove that if X is Compact, Y is T_2 and f:X→Y is one to one,onto and continous,thyen f is homeomorphism.

Assignment Question

MM 211 - Linear Algebra

1. For each of the following subsets of F^3, determine whether it is a subspace of F^3.
   a. $\{(x, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 0\}$
   b. $\{(x, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 4\}$
   c. $(x, x_2, x_3) \in F^3 : x_1 x_2 x_3 = 0$
   d. $(x, x_2, x_3) \in F^3 : x_1 = 5x_3$
2. Prove that if V is linearly independent in V, then so is the list $\{v_1 - v_2, v_2 - v_3, ..., v_{n-1} - v_n, v_n\}$
3. Let U be the subspace of R^5 defined by $U = \{(x_1, x_2, x_3, x_4, x_5)\} \in R^5 : x_1 = 3x_2 \;\&\; x_3 = 7x_4\}$ Find a basis of U.
4. Give an example of a function $f : R^2 \rightarrow R$ such that $f(av) = af(v)$ for all $a \in R$, and all $v \in R^2$, but f is not linear.
5. Prove that if T is a linear map F^4 to F^2 such that null $T = \{(x_1, x_2, x_3, x_4) \in F^4 : x_1 = 5x_2 \; and \; x_3 = 7x_4\}$ then T is surjective.
6. Prove that there does not exist a linear map from F^5 to F^2 whose null space equals $\{x_1, x_2, x_3, x_4, x_5 \in F^5 : x_1 = 3x_2 \;\&\; x_3 = x_4 = x_5\}$
7. Suppose that V is finite dimensional and $S, T \in L(V)$. Prove that ST is invertible if and only if both S and T are invertible.
8. Suppose $T \in L(V)$. Prove that if $U_1, U_2, ..., U_m$ are subspaces of V invariant under T, then $U_1 + U_2 + ... + U_m$ is invariant under T.
9. Define $T \in L(F^2)$ by $T(w, z) = (z, w)$. Find all eigen values and eigen values of T.
10. Find all eigenvalues and eigen vectors for $T \in L(F^3)$ by $T(z_1, z_2, z_3) = (2z_2 0, 5z_3)$
11. Find all generalized eigen vectors of $T \in L(C^2)$ defined by i. $T(w, z) = (z, v)$; (ii)) $T(w, z) = (-z, w)$
12. Give an example of an operator on C^4 whose characteristic polynomial equals i. $(z-7)^2$ ($(z-8)^2$; ii. $z(z-1)^2$
13. Prove that if V is an inner product space and $T \in \alpha(v)$ then trace $T* = tarace \; T$