

TOPICS FOR ASSIGNMENT

B.Sc MATHEMATICS (I-SEMESTER) (2017-2018)

MM1141 Core - I -Methods of Mathematics

1. Prove that in the set Q of rational numbers, the relation $R = \{(a, b) \sim (c, d) \text{ if } ad = bc\}$ is an equivalence relation.
2. Using mathematical induction prove that the sum of cubes of first n natural numbers is the square of $[\frac{n(n+1)}{2}]$.
3. Prove that there is no rational number $\frac{b}{a}$ whose square is 3.
4. State and prove Euclid's algorithm.
5. If $m \neq n$, then prove that $F(m)$ and $F(n)$ are relatively prime.
6. Find the g.c.d and find r and s so that $ar + bs = d$.
7. Show that if n is greater than 4, is not prime, then $(n - 1)! \equiv 0 \pmod{n}$
8. Show that $2560 \equiv 1 \pmod{56}$
9. Solve $313x \equiv 1 \pmod{453}$
10. Find the domain and range of the function $y = x\sqrt{9 - x^2}$
11. Sketch the graph of $y = -2(x + 1)^2 - 3$
12. Suppose that a car moves with a constant velocity of 88 feet per second in the direction of the s axis. Given that the coordinate and the car at time $t = 0$ is $s = 100$. Find an equation for s as a function t and graph the position versus time curve.
13. A spoked wheel with a diameter 3 ft rolls along a flat road without slipping. How far along the road does the wheel roll if the spokes turn through 225°
14. Find the graph of the parametric equation $x = \cos t, y = \sin t, (0 \leq t \leq 2\pi)$.
15. Evaluate
 - a) $\lim_{x \rightarrow \infty} [\sqrt{x^2 + ax} - \sqrt{x^2 + bx}]$
 - b) Assuming m and n are positive find $\lim_{x \rightarrow -\infty} \frac{2+3x^n}{1-x^m}$.
16. a) Show that $|x|$ is not differentiable at $x = 0$.
b) Find a formula for $f'(x)$.
17. a) find $\frac{dy}{dx}$ for $y = \sqrt{x - \sin^2 4x}$
b) find $\frac{d^2y}{dx^2}$ for $y = x \cos(5x) - \sin^2 x$
18. a) find all rational values of r such that $y = x^2$ satisfies the equation $3x^2y'' + 4xy' - 2y = 0$.
b) Use implicit differentiation to find the slope of the tangent line to the curve at the specified point $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4; (-1, \sqrt[3]{3})$
19. a) find the intervals on which $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ is increasing or decreasing.
b) find the inflection point if any $f(x) = (x - 2)^2$
c) A 13 ft ladder is leaning against a wall. If the top of the ladder slip down the wall at a rate of 2 ft/s, how fast will the foot be moving away from the wall when the top is 5 ft above the ground?
20. Find the relative extrema using both the first and second derivative test of
 - a) $f(x) = 1 - 4x - x^2$
 - b) Analyse the equation $y = \frac{2x^2 - 8}{x^2 - 16}$ Type equation here.

c) $y = 6\sqrt[3]{x} + 3\sqrt[3]{x}$ and analyse it

21) a) Let $s(t) = t^3 - 6t^2$ be the position function of a particle moving along the s -axis where s is in meters and t is in seconds

c) Find the equation of the hyperbola with vertices $(0, +8)$ and asymptotic $y = \frac{4}{3}x$.

22) a) Find the new coordinates of the point $(2, 4)$ if the coordinate axes are rotated through an angle of $\theta = 30^\circ$.

b) Identify and sketch the curve $15x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$

c) Find an equation for the parabola that satisfies the given coordinates vertex $(0, 0)$; focus $(3, 0)$.

23) i) State the well-ordering principle.

ii) State the Division algorithm.

iii) Give the numbers that are relatively prime.

iv) State Euclidean Algorithm.

v) State Bezout's identity.

vi) When two integers a and b are said to be congruent modulo.

vii) Define a power function with an example.

viii) Define the vertical asymptote of a function f .

ix) When $f(x)$ is said to be continuous at $x = 9$.

x) State the intermediate value theorem.

24) Evaluate

i) $\lim_{x \rightarrow 0} \frac{\tan 8x}{\sin 3x}$ ii) Find the slope of the curve $y = 2\sqrt{x}$ at $x = 4$

iii) Find an equation for the tangent line to the graph of $y = x^3\sqrt{(2-x^3)}$ at $x = 1$

iv) Find $d/dx[\cos(x^3)]$

v) Find dy/dx by implicit differentiation of $\frac{xy^3}{1+\sec y} = 1 + y^4$

25) a) A rocket rising vertically is tracked by a radar station is on the ground 5 minute from the launch pad. How fast is the rocket rising when its high and its distance from the radar station is increasing at a rate of 2000 .Find the instantaneous acceleration and show the graph of acceleration versus

b) Find the inflection point of $f(x) = \sin x$ on $[0, 2\pi]$ and confirm that results are consistent with the graph of the function.

c) Sketch the graph of the parabola

$$x^2 = -9y$$

d) Show that if $(a,m)=d, (b,m)=1$ then $(ab,m)=d$

e) If $m \neq n$, then prove that $F(m)$ and $F(n)$ are relatively prime.