Assignment Questions

M.Sc. Mathematics S4

Functional Analysis- II MM242

1) Show that an operator T on a Hilbert Sppace H is unitary iff T(e_i) is complete orthonormal set whenever $\{e_i\}$ is.
2) Prove that T_1 and T_2 are self adjoint operators on a Hilbert space H, prove that $T_1\, T_2$ +$T_2\, T_1$ is self adjoint
3) Let T be a normal operator on a finite dimensional Hilbert space H with spectrum $\{\lambda_{1,}, \lambda_{2,}, \dots \dots, \lambda_{m,}\}$. Then prove that

i) T is self - adjoint ⟺ each $\lambda_{i,}$,is real.

ii) T is positive ⟺ each $\lambda_i \geq 0$

iii) T is unitary ⟺ $|\lambda_i|$=1 for each i .

4) Show that the self adjoint operator is continuous map

5) Prove that a Hilbert space is seperable iff every ortho normal set in H is countable.

6) Show that an idem potent operator on aHilbert space H is a[projection on H iff it is normal.

7) Let $\{x_1 x_2 \dots \dots x_n,\}$ be an orthonormal set inX and $k_{1,} k_{2,.} \dots \dots \dots k_n$ be scalars having absolute value 1.Then $\|k_1 x_1 + k_2 x_2 + \cdots \dots k_n x_n\|$=$\|x_1 + \cdots \dots x_n\|$

8) Show that $r(x^n) = \left(r(x)\right)^n$; where $r(x)$ is the spectral radius

Assignment Questions

M.Sc . Mathematics S4 MM244

Analytic Number Theory

1) If $(a,b) = 1$, and if $^{c}/_{a}$ and $^{a}/_{b}$ then prove that (c,d)=1
2) If $(a,b) = 1$ and $^{d}/_{a+b}$.Prove that $(a,b) = (b,d)$=1
3) Given$x\ and\ \ y$,let $m = ax + by$ where $ad - bc \neq 1$.Prove that (m,n)=(x,y)
4) $(a,b) = 1$ show that $(a+b, a-b)$ is either 1 or 2
5) Prove that $\varphi(n)$ is an even integer
6) If n is an odd integer , then prove that $\varphi(2n)$=$\varphi(n)$
7) Find the value of the following

a)$19/23$

b) $20/31$

c) $8/11$

8) Find the order of integers 2,3and 5

a) modulo 17

b) modulo 19

9) Find all x such that $x \equiv 1(mod3)$

$x \equiv 2(mod4)$

$x \equiv 3(mod5)$

10) Prove that Diophantine equation $y^2 = x^3 + 11$ has no solution

10) Find (219/383)

11) Solve $25x \equiv 15(mod120)$

Assignment Questions

M.Sc . Mathematics S4

Complex Analysis II   MM241

1) Let U be a simply connected open set .Prove that there exists a holomorphic automorphism $f(z_1)$ =z_2 for $z_1, z_2 \in U$
2) Prove that $f(z)$=$\sum z^{n!}$ cannot be analytically contained to any open set strictly larger than the unit disc
3) Prove hat $|f(0)| \leq {}^1\!/_4$; whenever

$f\left({}^1\!/_2\right) = f\left({}^i\!/_2\right)$=0 and $|f(z)| \leq 1$;for all z in the closed unit disc such that f is analytic.

4) Prove that $\int_0^{2\pi} \log\left|e^{i\theta} - \frac{ae^{i\varphi}}{R}\right| d\theta = 0$

If $0 < a \leq R$

5) Let f be analytic in the neighborhood of a point $z_{0,}$, then P$\left(f_{\gamma,} Df_{\gamma,}, \ldots . Df_{\gamma,}\right)$=0 for D=$\frac{d}{dz}$ and f is continuous along a path γ
6) Let f be a meromorphic function which has only a finite number of poles in the unit disc; whenever $|f(z)|$=1; whenever$|z|$=1 .Prove that f is a rational function.
7) Let $\{U_n\}$be a sequence of harmonic function of harmonic functions on the open disc. Show that if it converges uniformly on compact subsets of the disc, then the limit is harmonic.
8) Show that (a) $\lim_{n\to\infty} \frac{n^2 \Gamma \mathrm{n}}{\Gamma(n+z)}$=1
9) Show that $\Gamma(z+1) = z\Gamma(z)$

10) Show that $\frac{1}{\Gamma(\mathrm{z})}$is an entire function of order 1

11) Show that $\prod \ (1+z_n)$converges absolutely iff $\prod \ (1+|z_n|)$ converges.

Assignment Questions

M.Sc . Mathematics S4

Coding Theory MM243

1. Let C be the code consisting of all words of length '4' having even weight. Determine the error pattern that C will correct.
2. Find the RREF for $\begin{bmatrix} 0 & 1 & 01 \\ 1 & 0 & 01 \\ 1 & 1 & 00 \end{bmatrix}$
3. Find a generator matrix for the code $C =< S >$ for $S = \{11000,01111, 11110, 01010\}$
4. Find the dimension of C for $C = \{000000, 001011, 010101, 100110, 101101, 110011, 111000\}$
5. Find a parity check matrix for $C =< S >; S = \{10101,01010,11111, 00011, 10110\}$
6. Find generator matrix for C and $C^{\perp}$ where $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
7. Decode (i) 110100 & (ii) 111111 for the PCM $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
8. Find a parity-check matrix for a linear code of length 'n' and with generator polynomial $g(x)$ for
   a. $n = 6\&g(x) = 1 + x^2$
   b. $n = 8; g(x) = 1 + x^2$
9. Let $g(x) = 1 + x^4 + x^6 + x^7 + x^8$; generates a 2- error correcting linear cyclic code C of length 15. Decode the following word.
   $$v = 001000001110110$$
10. If $h(x) = 1 + 1 + x^3 + x^5$; compute $f(x)$ mode $h(x)$, for $f(x) = x + x^4 + x^6 + x^7 + x^8$.
11. Find the minimal polynomial of $\alpha = \beta^3; \alpha \in GF(2^4)$ constructed using $h(x)$ =1+x+x^4

12.Construct a 2 error correcting BCH code C_{15} for $r = 4$ using $h(x) = 1 + \mathrm{x} + x^4$.