(Pages : 3) **G – 1462**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1641 : Real Analysis – II
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. Each carries **1** mark.

1. The function $g:\mathbb{R}\to\mathbb{R}$ defined by g(x) = 2x for x rational, and g(x) = x + 3 for x irrational is continuous at x = ________

2. Give an example for a function on [0, 1] that is discontinuous at every point of [0, 1] but | f | is continuous on [0, 1].

3. Find the points at which the function f(x) = | x | + | x + 1 | is not differentiable.

4. Using L'Hospital's Rule, find $\lim\limits_{x\to1}\frac{\ln x}{x-1}$.

5. Define a convex function on an interval $I\subseteq\mathbb{R}$.

6. Let $g(x)=\left|x^3\right|, x\in\mathbb{R}$. Find $g'(x)$ for $x\neq0$.

7. The norm of the partition P = (0, 1.5, 2, 3.4, 4) is ________

8. If $F(x)=\frac{1}{2}x^2$ for all $x\in[a,b]$, is the antiderivative of f on [a, b], evaluate $\int_a^b f$.

9. Define a step function.

10. If J = [c, d] is a subinterval of [a, b] and $\varphi_J(x)=1$ for $x\in J$ and $\varphi_J(x)=0$, elsewhere in [a, b], then evaluate $\int_a^b \varphi_J$.

P.T.O.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Show that the Dirichlet's function defined on $\mathbb{R}$ by, $f(x)=\begin{cases}1, & \text{if } x \text{ is rational}\\ 0, & \text{if } x \text{ is irrational}\end{cases}$ is not continuous at any point on $\mathbb{R}$.

12. If $I = [a, b]$ is a closed and bounded interval and $f : I \to \mathbb{R}$ is continuous on I, prove that f is bounded on I.

13. Show that the function $f(x) = \sin x$ is continuous on $\mathbb{R}$.

14. If $m \in \mathbb{Z}, n \in \mathbb{N}$ and $x > 0$, prove that $x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}}$.

15. If f and g are differentiable at c, show that f g is differentiable at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.

16. Find $f'(0)$, if $f(x)=\begin{cases}x^2 \sin\left(\frac{1}{x}\right), & \text{for } x \neq 0\\ 0 & \text{for } x = 0\end{cases}$.

17. If $f : I \to \mathbb{R}$ has a derivative at $c \in I$, show that f is continuous at c.

18. Explain briefly the tagged partition of a closed interval [a, b].

19. If $f, g \in R[a,b]$ show that $f + g \in R[a,b]$.

20. State and prove boundedness theorem for Riemann integral.

21. Show that the set Q_1 of rational numbers in [0, 1] is a null set.

22. If c is an interior point of an interval I at which $f : I \to \mathbb{R}$ has an extremum and if $f'(c)$ exists, then prove that $f'(c) = 0$.

SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. State and prove Maximum Minimum Theorem.

24. State and prove Caratheodry Theorem.

25. If $f(x)=\begin{cases}2, & 0\le x<1\\ 1, & \text{if } 1\le x\le 2\end{cases}$. Show that $f\in R[0, 2]$ and evaluate the integral.

26. State and prove squeeze theorem.

27. Let $I\subseteq \mathbb{R}$ be an interval and let $f : I\to\mathbb{R}$ be monotone on I. Then prove that the set of points $D\subseteq I$ at which f is discontinuous is a countable set.

28. State and prove Rolle's Theorem.

29. Let $g:\mathbb{R}\to\mathbb{R}$ be defined by $g(x)=x+2x^2\sin\left(\frac{1}{x}\right)$, for $x\neq 0$ and $g(0) = 0$.

    Show that g is not monotonic in any neighborhood of 0.

30. Prove that $1-\frac{1}{2}x^2\le \cos x$ for all $x\in\mathbb{R}$.

31. If F, G are differentiable on [a, b], and $f=F', g=G'$ belongs to R[a, b], then prove that $\int_a^b fG = FG/_a^b - \int_a^b Fg$.

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) State and prove location of roots theorem.

    b) State and prove continuous inverse theorem.

33. a) State and prove Cauchy criterion for Riemann integrability.

    b) If $f : [a, b]\to\mathbb{R}$ is continuous on [a, b], then prove that $f\in R[a, b]$.

34. a) State and prove additivity theorem.

    b) If f and g are Riemann Integrable prove that fg is Riemann Integrable.

35. a) Let I be an open interval and let $f : I\to\mathbb{R}$ have a second derivative on I. Then prove that f is a convex function if and only if $f''(x)\ge 0$ for all $x\in I$.

    b) Use Newton's Method to find an approximate value of $\sqrt{2}$.

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(Pages : 3) **G – 1464**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme Under CBCSS
Mathematics
Core Course – XI
MM 1643 – COMPLEX ANALYSIS – II
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory. Each** carries **1** mark.

1. Write the power series expansion of $f(z)=\frac{1}{1+z}$ in the disc $|z|<1$.

2. Test whether the series $\sum_{n=0}^{\infty}\frac{z^n}{n!}$ is convergent in C.

3. What are the singular points of $ze^{\frac{1}{z}}$?

4. Define essential singularity for a function f(z) at z = a.

5. Describe the nature of singularity for $f(z)=\frac{1-e^{2z}}{z^4}$ at z = 0.

6. What is the order of the pole for $f(z)=\frac{1}{(2\sin z-1)^2}$?

7. Find the residue of $f(z) = \frac{e^z}{z^2}$ at its singularity.

8. What is the residue of $f(z)=\frac{1}{z}+1+z+z^2+\ldots$ at z = 0 ?

9. What is the order of the zero of $z(e^z - 1)$?

10. If f is an even function, then what is the value of $\int_{-\infty}^{\infty} f(x)dx$?

P.T.O.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Find a power series expansion for $f(z) = \frac{z-1}{z+1}$ about z = 0.

12. Prove that $\int_C \frac{\sin z}{\left(z - \frac{\pi}{2}\right)^2} dz$, where C is the circle |z| = 2.

13. Evaluate $\int_C \frac{e^{iz}}{z} dz$ where C is |z| = 2.

14. Determine and identify the singularities of $\frac{z}{1+z^2}$.

15. Determine the order of the pole and residue at z = 0 for $\frac{\sinh z}{z^4}$.

16. Find the residue of cot z at z = 0.

17. Write the principal part of the function $f(z) = z \exp\left(\frac{1}{z}\right)$ at its isolated singular point and determine the value of the singularity.

18. If a is a zero of order r for $\frac{1}{f(z)}$, then prove that a is a pole of order r for f(z).

19. Find the residue of $\frac{1}{\left(z^2+a^2\right)^2}$ at z = ai.

20. State Jordan's Lemma.

21. Find the residue of $\frac{ze^z}{(z-1)^3}$ at its pole.

22. Find $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

SECTION – III

Answer **any six** questions from this Section. **Each** question carries **4** marks.

23. State and prove Cauchy's theorem.

24. Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle |z| = 2.

25. Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2} dz$ where C is |z| = 3.

26. Let f be analytic inside and on a simple closed curve C. Then prove that

$f'(z) = \frac{1}{2\pi i}\int_C \frac{f(\xi)}{(\xi - z)^2}\, d\xi$, where z is any point inside C.

27. Let f be a function which is bounded and analytic throughout a domain $0 < |z - z_0| < \delta$. Then prove that either f is analytic at z_0 or else z_0 is a removable singular point of δ.

28. Prove that an isolated singularity a of f(z) is a pole if and only if $\lim_{z \to a} f(z) = \infty$.

29. Prove that $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}$.

30. Prove that $\int_0^{\pi} \frac{a\,d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{\sqrt{a^2+1}}$, a > 0.

31. Find $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{5^n}$.

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) State and prove Cauchy's integral formula. 7

b) Find the residue of $\frac{e^z}{z^2(z^2+9)}$ at its poles. 8

33. a) State and prove Casorati-Weierstarss theorem. 7

b) Prove that $\int_{-\infty}^{\infty} \frac{(x^2 - x + 2)}{x^4 + 10x^2 + 9}\, dx = \frac{5\pi}{12}$. 8

34. a) Prove that $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2} = \frac{\pi}{4a^3}$. 8

b) Show that $\int_0^{\infty} \frac{\sin x}{x}\, dx = \frac{\pi}{2}$. 7

35. a) Use residue, evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}$, $-1 < a < 1$. 8

b) Prove that $\int_0^{\infty} \frac{\cos x}{1+x^2}\, dx = \frac{\pi}{2e}$. 7

(Pages : 4) **G – 1465**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
Core Course – XII
MM 1644 : Abstract Algebra – II
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. **Each** carries **1** mark.

1. Find $\phi(25)$ for the homomorphism $\phi : \mathbb{Z} \to \mathbb{Z}_7$ such that $\phi(1) = 4$.

2. How many homomorphisms are there of $\mathbb{Z}$ into $\mathbb{Z}$?

3. Find the order of the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_2)/\langle(2, 1)\rangle$.

4. The trivial subgroup N = {0} of $\mathbb{Z}$ is a normal subgroup. Compute $\mathbb{Z}/\{0\}$.

5. The image of a group of 6 elements under a homomorphism may have 12 elements. True or False.

6. Compute the product (– 3, 5) (2, – 4) in the ring $\mathbb{Z}_4 \times \mathbb{Z}_{11}$.

7. Find all units in the ring $\mathbb{Z} \times \mathbb{Z}$.

8. Find the characteristic of the ring $\mathbb{Z}_3 \times \mathbb{Z}_3$.

9. Using Fermat's theorem, find the remainder of 3^{47} when it is divided by 23.

10. A ring homomorphism $\phi : R \to R'$ carries ideals of R into ideals of R'. True or False.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Show that a group homomorphism $\phi : G \to G'$ is a one-to-one map if and only if $Ker(\phi) = \{e\}$.

12. Let H be a normal subgroup of G. Then show that $\gamma : G \to G/H$ given by $\gamma(x) = xH$ is a homomorphism with kernel H.

13. Does there exist a nontrivial homomorphism $\phi : \mathbb{Z}_3 \to \mathbb{Z}$? If yes, give an example. If not, explain why that is so.

14. Show that any group homomorphism $\phi : G \to G'$ where $|G|$ is a prime must either be the trivial homomorphism or a one-to-one map.

15. Show that a factor group of a cyclic group is cyclic.

16. Let (R, +) be an abelian group. Show that (R, +, .) is a ring if we define ab = 0 for all $a, b \in R$.

17. Are the fields $\mathbb{R}$ and $\mathbb{C}$ isomorphic ? Justify your answer.

18. In the ring $\mathbb{Z}_n$, show that the divisors of 0 are precisely those nonzero elements that are not relatively prime to n.

19. Show that 1 and p – 1 are the only elements of the field $\mathbb{Z}_p$ that are their own multiplicative inverse.

20. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and having derivatives of all orders. Differentiation gives a map $\delta : F \to F$ where $\delta(f(x)) = f'(x)$. Is δ a homomorphism ? Why ?

21. Show that each homomorphism from a field to a ring is either one to one or maps everything onto 0.

22. Show that if R is a ring with unity and N is an ideal of R such that $N \neq R$, then R/N is a ring with unity.

SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. Let $\phi : G \to G'$ be a group homomorphism. Show that if $|G|$ is finite, then $|\phi[G]|$ is finite and is a divisor of $|G|$.

24. Show that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G.

25. Show that an intersection of normal subgroups of a group G is again a normal subgroup of G.

26. Show that if U is the collection of all units in a ring (R, +, .) with unity, then (U, .) is a group.

27. Show that every finite integral domain is a field.

28. Find all positive integers n such that $\mathbb{Z}_n$ contains a subring isomorphic to $\mathbb{Z}_2$.

29. Find all solutions of the congruence $155x \equiv 75 \pmod{65}$.

30. Let R be a commutative ring with unity of prime characteristic p. Show that the map $\phi_p : R \to R$ given by $\phi_p(a) = a^p$ is a homomorphism.

31. A ring R is a Boolean ring if $a^2 = a$ for all $a \in R$. Show that every Boolean ring is commutative.

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) Prove or disprove : If d divides the order of G, then there must exist a subgroup H of G having order d. 10

    b) Let ϕ be a homomorphism of a group G into a group G'. If K' is a subgroup of G', then show that $\phi^{-1}[K']$ is a subgroup of G. 5

33. a) Let $\phi : G \to G'$ be a homomorphism with kernel H and let $a \in G$. Prove the set $\{x \in G \mid \phi(x) = \phi(a)\} = Ha$. 5

b) Let H be a normal subgroup of G. Show that the cosets of H form a group G/H under the binary operation (aH) (bH) = (ab)H. 5

c) Show that if H and N are subgroups of a group G, and N is normal in G, then $H \cap N$ is normal in H. Show by an example that $H \cap N$ need not be normal in G. 5

34. a) An element a of a ring R is idempotent if $a^2 = a$. Find all idempotents in the ring $\mathbb{Z}_6 \times \mathbb{Z}_{12}$. 5

b) Show that the unity element in a subfield of a field must be the unity of the whole field. 5

c) Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$. 5

35. a) Show that a division ring contains exactly two idempotent elements. 5

b) Show that the characteristic of a subdomain of an integral domain D is equal to the characteristic of D. 5

c) Show that $2^{11,213} - 1$ is not divisible by 11. 5

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(Pages : 4) **G – 1466**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
Elective
MM 1661.1 : Graph Theory
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

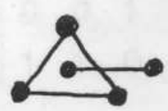
1. Define a simple graph.

2. The number of odd vertices in a graph is always ________

3. What is a spanning subgraph ?

4. Define outdegree.

5. Is the following graph connected ?



6. Define Euler graph.

7. What is a unicursal graph ?

8. Define radius of a graph.

9. A tree with n vertices has ________ edges.

10. What is maximal tree of a graph ?

SECTION – II

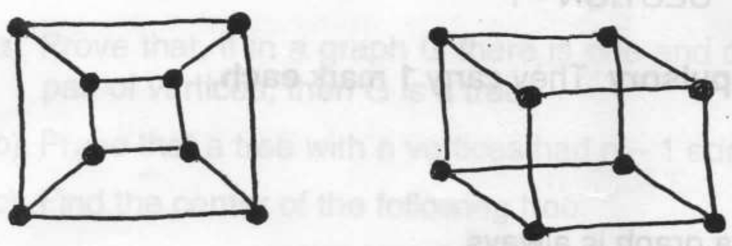
Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Write any two applications of graph theory with suitable explanation.

12. Write the adjacency matrix of C_4.

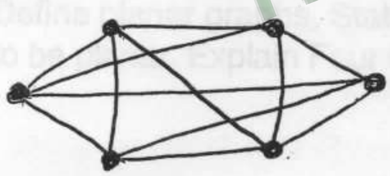
13. Prove that the sum of degrees is equal to twice the number of edges.

14. Label the following graphs to prove that they are isomorphic.



15. Prove that a graph G is disconnected if and only if the vertex set can be partitioned into 2 non-empty disjoint subsets V_1 and V_2 such that there is no edge having one end vertex in V_1 and another in V_2.

16. Is the following graph Euler graph ? Explain.



17. Explain Chinese Postman problem.

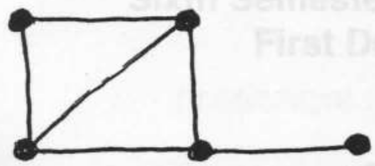
18. State a characterization theorem for Euler digraph. Illustrate with an example.

19. Prove that there is one and only one path between every pair of vertices in a tree T.

20. Prove that a graph with n vertices, n – 1 edges and no circuits is connected.

21. Prove that a graph G is a tree if and only if it is minimally connected.

22. Define spanning tree. Find a spanning tree of the following graph.



SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Draw all non-isomorphic graphs on 4 vertices. How many of them are self-complementary ? How many are connected ?

24. Define spanning subgraph and induced subgraph. Is P_4 a spanning subgraph of K_4 ? Is it an induced subgraph ? Explain.

25. Define incidence matrix. Draw the graph with incidence matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

26. Prove that a graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ components.

27. In a connected graph G with exactly 2k odd vertices, prove that there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

28. Prove that every tree has either one or two centers.

29. Prove that every connected graph has at least one spanning tree.

30. Draw planar representations of K_4 and a cube.

31. Prove that in any simple connected planar graph with f regions, n vertices and e edges, $e \geq \frac{3f}{2}$ and $e \leq 3n - 6$.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

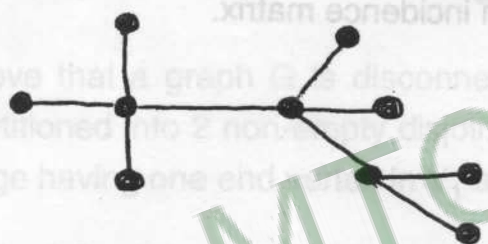
32. Explain in detail multicolour cube puzzle and its graph theoretic model.

33. Explain decanting problem with its graph theoretic formation.

34. a) Prove that, if in a graph G there is one and only one path between every pair of vertices, then G is a tree.

    b) Prove that a tree with n vertices had n – 1 edges.

    c) Find the center of the following tree.



35. Define planar graphs. State a necessary and sufficient condition for a graph G to be planar. Explain Four Colour Theorem and its graph theoretic interpretation.